EE 440 Photonic Systems & Technologies 2025 – Série 3

SOLUTIONS

Exercise 1

A 1.55 um lightwave systems operating at 5 Gb/s is using 100 ps (FWHM) Gaussian pulses chirped such that $C = -6$. Neglect laser linewidth and assume that $\beta_2 = 20\ \mathrm{ps^2/nm}$.

(a) What is the dispersion limited maximum fiber length using the criteria $4B\sigma_{max} \leq 1$?

We use the following expression:

$$\frac{\sigma^2(z)}{\sigma_0^2} = \left(1 + \frac{C\beta_2 z}{2\sigma_0^2}\right)^2 + (1 + V_\omega^2)\left(\frac{\beta_2 z}{2\sigma_0^2}\right)^2 + (1 + C^2 + V_\omega^2)^2\left(\frac{\beta_3 z}{4\sqrt{2}\sigma_0^3}\right)^2$$

We are given that $T_{FWHM} = 100\ \mathrm{ps}$, we therefore have for our Gaussian pulse that $\sigma_0 = 42.47\ \mathrm{ps}$.

Moreover the maximum RMS width that can be tolerated is $\sigma_{max} = \frac{1}{4B} = 50\ \mathrm{ps}$.

For the 100-ps-duration pulse with $V_\omega \ll 1$ and the β_3 ignored, theequation becomes:

$$1.386 = \left(1 - \frac{6(20)z}{2(42.47)^2}\right)^2 + \left(\frac{20z}{2(42.47)^2}\right)^2$$

$$1.386 = (1 - 0.33z)^2 + (0.0055z)^2$$

$$(1.12 \cdot 10^{-3})z^2 - 0.066z - 0.386 = 0$$

The solution to the quadratic equation for the distance , which must be positive is:

$$z \approx 64\ \mathrm{km}$$

(b) How much will it change if the pulses were unchirped?

For an unchirped pulse, we set C = 0. The equation becomes:

$$1.386 = 1 + (3.07 \cdot 10^{-5})z^2$$

And the solution is

$$z \approx 112\mathrm{km}$$

Unchirped case can propagate longer.

Exercise 2

Starting from the definition for the group velocity $v_g = \frac{d\omega}{d\beta} = \frac{c}{n_g}$ prove that the group index can be expressed as a function of wavelength as $n_g(\lambda) = n(\lambda) - \lambda\frac{dn}{d\lambda}$.

We use the fact that $\beta = n(\lambda)\frac{2\pi}{\lambda}$ and that $\lambda = \frac{2\pi c}{\omega}$. We have :

$$n_g = c\frac{d\beta}{d\omega} = c\frac{d\beta}{d\lambda}\frac{d\lambda}{d\omega}$$

$$n_g(\lambda) = c\left[-\frac{2\pi}{\lambda^2}n(\lambda) + \frac{2\pi}{\lambda}\frac{dn}{d\lambda}\right]\left[-\frac{2\pi c}{\omega^2}\right]$$

$$n_g(\lambda) = c\frac{2\pi}{\lambda}\left[\frac{dn}{d\lambda} - \frac{1}{\lambda}n(\lambda)\right]\left[-2\pi c\frac{\lambda^2}{(2\pi c)^2}\right]$$

$$n_g(\lambda) = c\frac{2\pi}{\lambda}\left[\frac{1}{\lambda}n(\lambda) - \frac{dn}{d\lambda}\right]\left[\frac{\lambda^2}{2\pi c}\right]$$

$$n_g(\lambda) = n(\lambda) - \lambda\frac{dn}{d\lambda}$$

Exercise 3

Let us consider a fiber made of the glass material fused silica SiO2. The refractive index can be modeled using Sellmeier relation and the following coefficients:

$$n^2 = 1 + \sum_{j=1}^{3}\frac{A_j\lambda^2}{\lambda^2 - \lambda_j^2}$$

$$A_1 = 0.6961663$$

$$\lambda_1 = 0.0684043\ \mu\text{m}$$

$$A_2 = 0.4079426$$

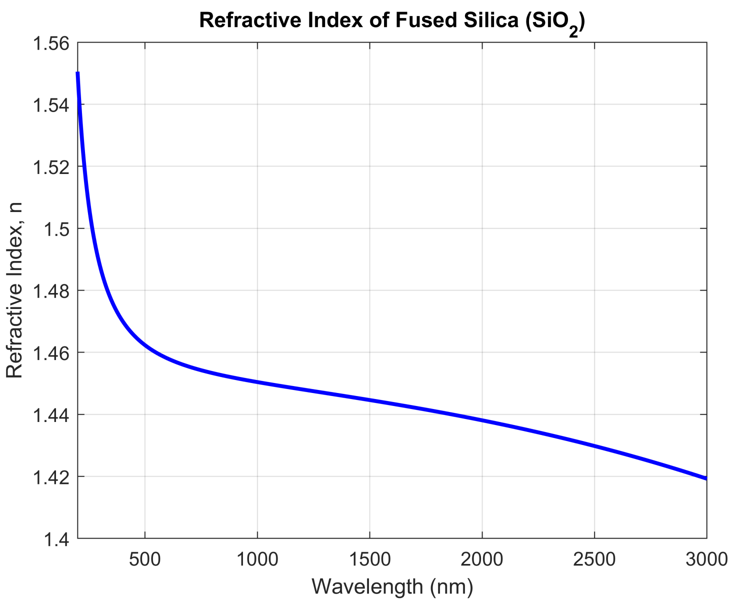
$$\lambda_2 = 0.1162414\ \mu\text{m}$$

$$A_4 = 0.8974794$$

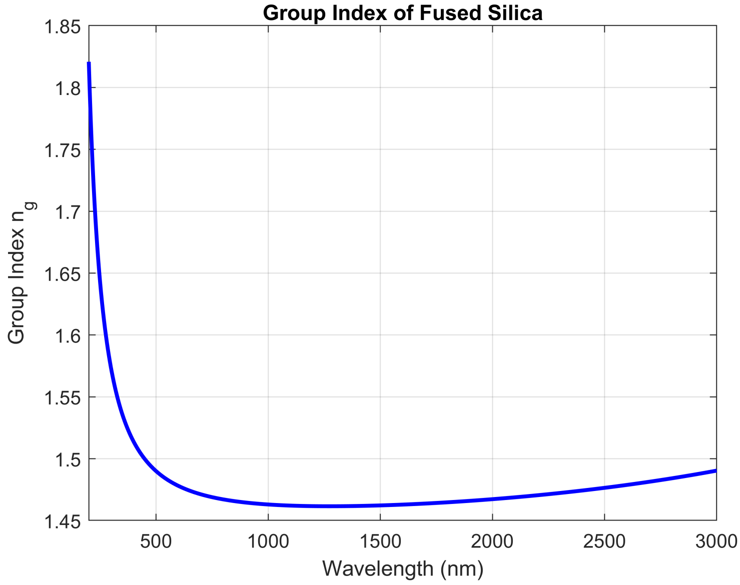
$$\lambda_4 = 9.896161\ \mu\text{m}$$

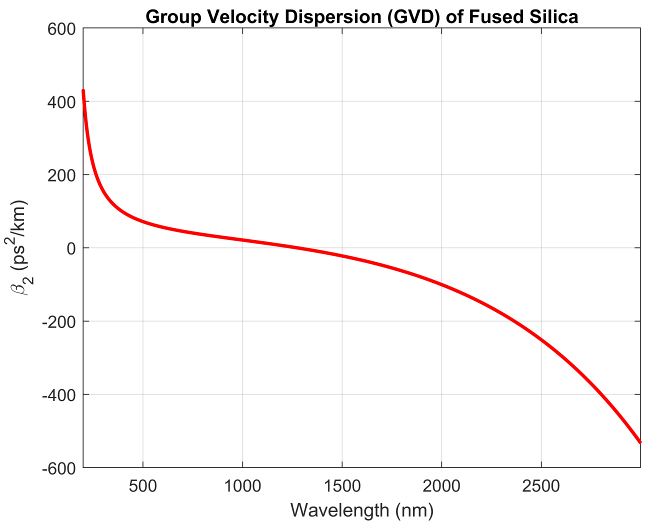
Using matlab:

(a) Plot $n(\lambda)$ from 200 nm to 3000 nm



(b) Plot the group index n_g and the group velocity dispersion β_2. It is preferable to base the derivation on an equally spaced **frequency** grid. Remember: that the group index is most conveniently written in terms of frequency and is given by $n_g = \frac{d\beta}{d\omega}c$, with c the speed of light. And that the GVD is given by $\beta_2 = \frac{d^2\beta}{d\omega^2}$





Exercise 4

A 1.06 µm pulsed Q-switched Nd:YAG laser emits an un-chirped Gaussian pulse with 1 nJ energy and 100 ps FWHM. The pulses are transmitted through a 1 km long fiber with 3 dB/km loss, $n_2 = 3 \cdot 10^{-20}$ m^2/W and an effective area of 20 µm^2. Assume 94% of the energy of a Gaussian is within its FMHW.

(a) Calculate the maximum values of the nonlinear phase shift at the fiber output.

The maximum nonlinear phase shift due to peak power P_0 at the fiber output with effective length L_{eff} and nonlinear coefficient γ is obtained by

$$\varphi_{NL}(max) = \gamma\, P_0 L_{eff}$$

The effective length of 1 km ling fiber with 3 dB/km attenuation factor is $L_{eff} = \frac{1-10^{-\alpha_{\mathrm{dB/km}} \cdot L/10}}{\alpha_{\mathrm{dB/km}}/4.343} = 722\ m.$

As 94% of the total pulse energy is within FWHM, a pulse with 1 nJ energy and 100 ps FWHM has a peak power of $P_0 = 9.4\ W$.

Besides the nonlinear coefficient for a fiber with effective area $A_{eff} = 20\ \mu m^2$ and $n_2 = 3 \times 10^{-20}\ m^2/W$ operating at $\lambda = 1.06\ \mu m$ is $\gamma = \frac{2\pi n_2}{\lambda_0 A_{\mathrm{eff}}} = 8.9\ W/km$.

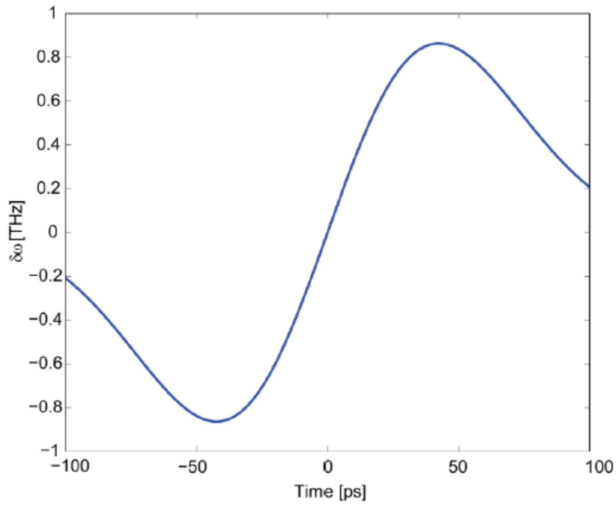
Therefore, the maximum nonlinear phase shift would become $\boldsymbol{\varphi_{\mathrm{NL}}(\mathrm{max}) = 60.34\ \mathrm{rad}}$.

(b) Express the frequency chirp at the fiber output. Plot it using matlab. What is the maximum value of the frequency chirp (in THz) and at which time (relative to the pulse center) does it occur ?

The frequency chirp $\delta\omega(t)$ due to self phase modulation for a Gaussian pulse with amplitude $A(t) = \sqrt{P_0}\exp\left[-0.5\left(\frac{t}{T_0}\right)^2\right]$ can be obtained by:

$$\delta\omega(t) = -\frac{\partial \phi_{\mathrm{NL}}}{\partial T} = -(\gamma P_0 L_{\mathrm{eff}})\frac{\partial}{\partial T}|U(0,T)|^2 = 2\gamma P_0 L_{eff}\frac{t}{T_0}\exp\left[-\left(\frac{t}{T_0}\right)^2\right]$$

This equation shows $\delta\omega(t)$ is a function of time therefore it can be demonstrated along pulse duration in the figure below. Note that for the input Gaussian pulse, T_{FWHM} and T_0 are related by $T_{FWHM} = T_0 2\sqrt{\ln(2)}$, thus we can calculate $T_0 = 60\ ps$.



It can be shown from figure (also by derivation of $\delta\omega(t)$), that the maximum frequency chirp is experienced at $\boldsymbol{t = \pm 42.4\ ps}$ leading to $\boldsymbol{\delta\omega_{max} = \pm 0.86\ THz}$.

Exercise 5

We have a 50 km long fiber link. The radius of the fiber is 2.5 μm and the peak Raman coefficient is $g_R = 2 \cdot 10^{-13}$ m/W. The loss at 1.3 μm and 1.5 μm is 0.5 dB/km and 0.2 dB/km, respectively.

(a) Calculate the threshold power for stimulated Raman scattering if the operating wavelength is 1.3 μm.

The threshold power of stimulated Raman scattering is approximated by:

$$P_{th} = \frac{16A_{\text{eff}}}{g_r L_{\text{eff}}}$$

where g_r, $A_{\text{eff}}$and L_{eff} are respectively peak Raman coefficient, effective area and effective surface. The effective length is calculated from (1-exp(-αL))/α with α being the linear loss coefficient at the pump wavelength.

For the operating wavelength at 1.3 µm with 0.5 dB/km loss and assuming fiber radius of 2.5 µm and peak Raman coefficient $g_r = 2\times10^{-13}$ m/W we obtain Raman threshold as: $\boldsymbol{P_{th}}$**= 181.4 mW**.

(b) How much does the threshold power change if the link operating wavelength is changed to 1.5 μm.

Assuming that the effective area and the raman gain do not significantly change, by operating at 1.5 µm with 0.2 dB/km loss, the Raman threshold is reduced to $\boldsymbol{P_{th}}$**= 80.4 mW.**

Graded Exercise

The pulse broadening equation, where σ_0 is the initial RMS pulse width and σ the RMS pulse width after a propagating distance of L, is given in equation 1. For a RMS spectral width of σ_ω, V_ω is defined as $V_\omega = 2\sigma_\omega\sigma_0$.

$$\frac{\sigma^2(z)}{\sigma_0^2} = \left(1 + \frac{C\beta_2 z}{2\sigma_0^2}\right)^2 + (1 + V_\omega^2)\left(\frac{\beta_2 z}{2\sigma_0^2}\right)^2 + (1 + C^2 + V_\omega^2)^2\left(\frac{\beta_3 z}{4\sqrt{2}\sigma_0^3}\right)^2$$

We have the following operating conditions: small spectral width, no chirp, zero-dispersion wavelength operation (λ close to 1320 nm)

(a) Simplify equation 1 according to the operating conditions.

Small spectral width: $V_\omega \ll 1$

No chirp $C = 0$

Zero dispersion wavelength operation: $\beta_2 = 0$

We therefore get :

$$\frac{\sigma^2(z)}{\sigma_0^2} = 1 + \left(\frac{\beta_3 z}{4\sqrt{2}\sigma_0^3}\right)^2$$

$$\frac{\sigma(z)}{\sigma_0} = \sqrt{1 + \frac{1}{2}\left(\frac{\beta_3 z}{4\sigma_0^3}\right)^2}$$

(b) Find the input pulse width condition $\sigma_{0,min}$ for which σ is minimized. What is σ in this case (it should be a function of β_3 and L)?

We find that the condition, by setting the derivative to zero, is achieved for

$$\sigma_{0,min} = \left(\frac{|\beta_3|L}{4}\right)^{\frac{1}{3}}$$

We plug this back in the previous expression and find that:

$$\sigma = \left[\sigma_0^2 + \frac{\sigma_0^2}{2}\right]^{\frac{1}{2}}$$

$$\sigma = \sigma_0 \left[\frac{3}{2}\right]^{\frac{1}{2}}$$

$$\sigma = \sqrt{\frac{3}{2}}\left(\frac{|\beta_3|L}{4}\right)^{\frac{1}{3}}$$

(c) Given that the constraint on RMS pulse width after propagation relative to the bit width is given by $\sigma \leq 1/(4B)$, and using s derived in b), express the BL limit for this system

The constraint is given as σ ≤ 1/(4B). Using the expression from the previous part we get :

$$\sigma = \sqrt{\frac{3}{2}}\left(\frac{|\beta_3|L}{4}\right)^{\frac{1}{3}} \leq \frac{1}{4B}$$

$$B(|\beta_3|L)^{\frac{1}{3}} \leq \frac{(4)^{\frac{1}{3}}}{4}\sqrt{\frac{3}{2}}$$

$$B(|\beta_3|L)^{\frac{1}{3}} \leq 0.324$$

(d) Using the derived equation, what is the limiting bit rate for $L = 100\ \text{km}$ and slope $S = 0.117\ \text{ps}/(\text{nm}^2\text{km})$.

We first calculate β_3 from the slope and get and β_3 = 0.1 ps^3/km. We then get that the limiting bit rate is :

$$B(0.117 \times 10^{-36} \times 100)^{1/3} \leq 0.324$$

$$B \leq \frac{0.324}{(0.117 \times 10^{-36} \times 100)^{1/3}}$$

$$B \leq 142.7 \text{ Gb/s}$$